CALCULATION OF NONSTEADY CONVECTIVE HEAT TRANSFER FOR TURBULENT VISCOUS INCOMPRESSIBLE FLOW IN A TUBE OF ELLIPTICAL CROSS SECTION

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An approximate analytical expression is obtained for the temperature field. The variations of the dimensionless mass-average temperature of the fluid and the dimensionless integral-average heat flux at the tube wall are determined for various values of the Reynolds and Prandtl numbers.

We consider the problem of determining the nonsteady temperature field in viscous, hydrodynamically stabilized, tubulent flow of a fluid in a semiinfinite tube of elliptical cross section. We assume that the flow is quasisteady, the fluid is incompressible, its physical properties do not depend on the temperature, and the variation of the heat-flux density in the axial direction due to heat conduction is small in comparison with the variation due to convection.

The fluid has a constant temperature T_0 at the initial time $\tau = 0$ and at the tube entry $z = 0$. Beginning at time $\tau = 0^+$ the inner surface of the tubewall is maintained at a constant temperature $T_W = T_0$.

The problem of determining the temperature field is reducible to the solution of the nonsteady energy equation in dimensionless form

$$\frac{\partial\Theta}{\partial \mathrm{Fo}} + U(X, Y)\frac{\partial\Theta}{\partial Z} = \frac{\partial}{\partial X}\left[\left(1 + \frac{\mathrm{Pr}}{\mathrm{Pr}_m}\bar{\nu}\right)\frac{\partial\Theta}{\partial X}\right] + \frac{\partial}{\partial Y}\left[\left(1 + \frac{\mathrm{Pr}}{\mathrm{Pr}_m}\bar{\nu}\right)\frac{\partial\Theta}{\partial Y}\right] \tag{1}$$

$$(\mathrm{Fo} > 0,\ Z > 0,\ |X| < \sqrt{2 - e^2},\ |Y| < \sqrt{(2 - e^2 - X^2)(1 - e^2)}\ )$$

subject to the boundary and initial conditions

$$\left.\begin{aligned} &\Theta(X, Y, Z, \mathrm{Fo})|_{\mathrm{Fo}=0} = 0, \\ &\Theta(X, Y, Z, \mathrm{Fo})|_{Z=0} = 0, \\ &\Theta(X, Y, Z, \mathrm{Fo})|_{S} = 1. \end{aligned}\right\} \tag{2}$$

Here $\Theta = (T - T_0)/(T_W - T_0)$; $X = x/R$; $Y = y/R$; $Z = z/\mathrm{Pe}\cdot R$; $\mathrm{Pe} = u_* R/a$; $\mathrm{Fo} = a\tau/R^2$; $R^2 = b^2c^2/(b^2 + c^2)$; $U = \omega_Z/u_*$; $T(x, y, z, \tau)$, unknown temperature field; x, y, z, Cartesian coordinates; τ, time; ω_Z, velocity profile of the turbulent fluid flow in the duct; a, thermal diffusivity of the fluid; Pr, Prandtl number; Pr_m, turbulent Prandtl number ($\mathrm{Pr}_m \approx 1$ [1]); $\bar{\nu}$, turbulent viscosity coefficient in the fluid flow; $u_* = \sqrt{\tau_W/\rho}$, average dynamic velocity of the fluid; $\bar{\tau}_W$, perimeter-average tangential stress on the tube wall; and S, index referring to the lateral surface of the tube. Everywhere Θ, U, $\bar{\nu}$ represent the dimensionless average values of the temperature, velocity, and viscosity of the fluid, respectively, and $e^2 = 1 - (b/c)^2$.

In determining the profile of the velocity U(X, Y) and turbulent viscosity $\bar{\nu}$ we use the following expression derived in [2]:

$$U(X, Y) = 2.5\ln\left(1 + \frac{1.2W}{3 - 2W/W_{\max}}\right) + 7.8\left(1 - \exp\left(-\frac{W}{\beta}\right) - \right.$$
$$\left. - \exp\left(-\frac{W}{\beta}0.33W\right)\right);\ \bar{\nu}(X, Y) = \frac{\varkappa}{3}\left(W - \beta\,\mathrm{th}\frac{W}{\beta}\right)(3 - 2W/W_{\max}),$$

where $\varkappa = 0.423$; $\beta = 11$; $W = \mathrm{Re}_*\tilde{R}\tilde{W}$; $\tilde{R} = R\Pi/f$; $\mathrm{Re}_* = u_*R/\nu$ is the dynamic Reynolds number; $\tilde{W}(X, Y)$ is the solution of the Poisson equations

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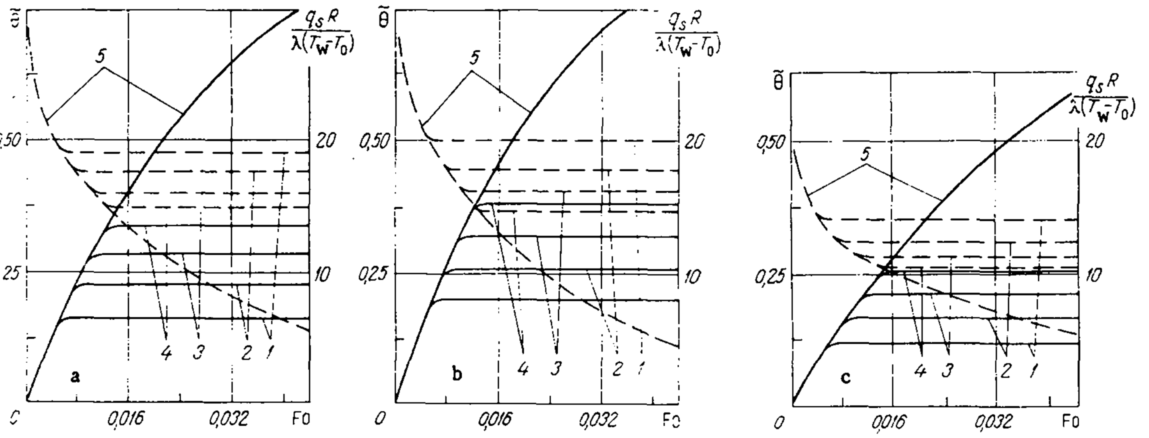

Fig. 1. Variation of mass-average fluid temperature (solid curves) and dimensionless integral-average heat flux at the tube wall (dashed curves). a) $Re_* = 500$, $Pr = 0.7$; b) 500, 1; c) 300, 0.7; 1) Z = 0.1; 2) 0.15; 3) 0.2; 4) 0.25; 5) 7.0.

$$\frac{\partial^2 \tilde{W}}{\partial X^2} + \frac{\partial^2 \tilde{W}}{\partial Y^2} = -1, \tag{3}$$

$$\tilde{W}(X, Y)|_S = 0 \tag{4}$$

for a domain of elliptical cross section; Π and f are the perimeter and cross-sectional area of the tube; and $W_{max} = Re_* \tilde{R} \tilde{W}_{max}$. The foregoing expressions for U(X, Y) and $\bar{\nu}$(X, Y) are based on formulas obtained by Reichardt [8] for the velocity profile and turbulent viscosity.

The expression for $\tilde{W}$(X, Y) has the form [3]

$$\tilde{W}(X, Y) = [2 - e^2 - X^2 - (1 - e^2) Y^2]/(4 - 2e^2).$$

To solve problem (1), (2) we apply the Bubnov–Galerkin (BG) method [4] and the method of characteristics [5] simultaneously. We seek an approximate solution in the form

$$\Theta_n(X, Y, Z, \mathrm{Fo}) = 1 + \sum_{k=1}^{n} a_k(Z, \mathrm{Fo})\, \Phi_k(X, Y),$$

where

$$\Phi_k(X, Y) = \frac{U(X, Y)}{(2 - e^2)^{k-1}} [X^2 + (1 - e^2) Y^2]^{k-1}, \quad k = 1, 2, \ldots, n,$$

is a system of coordinate functions satisfying the requirements of the BG method [4] and a_k(Z, Fo) denotes unknown functions, which are determined from a system of linear homogeneous first-order partial differential equations [6] by the method of characteristics.

We obtain the solution of problem (1), (2) in the second approximation (n = 2) in the form

$$\Theta_2(X, Y, Z, \mathrm{Fo}) = 1 + \sum_{k=1}^{2} \Phi_k(X, Y) \begin{cases} a_k^1(\mathrm{Fo}), & Z > \mu_1 \mathrm{Fo}, \\ a_k^2(Z, \mathrm{Fo}), & \mu_2 \mathrm{Fo} < Z < \mu_1 \mathrm{Fo}, \\ a_k^3(Z), & Z < \mu_2 \mathrm{Fo}, \end{cases} \tag{5}$$

where a_k^1(Fo); a_k^2(Z, Fo); a_k^3(Z); μ_k (k = 1, 2) are defined in [7].

To facilitate the computations we introduce the dimensionless mass-average temperature of the fluid [1] and the integral-average heat flux at the tube wall [3]:

$$\tilde{\Theta}(Z, \mathrm{Fo}) = \frac{\iint_D U(X, Y)\, \Theta_2(X, Y, Z, \mathrm{Fo})\, dD}{\iint_D U(X, Y)\, dD}, \tag{6}$$

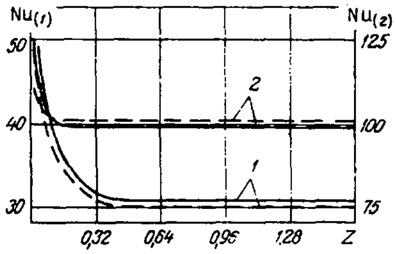

Fig. 2. Variation of local Nusselt number Nu at tube wall, calculated according to (5) (dashed curves) and the equation in [1] (solid curves) for Pr = 0.7. 1) Re = 10^4; 2) Re = $5 \cdot 10^4$.

$$\tilde{q}_S(Z,\ \text{Fo}) = \int_S q_S dS/\Pi, \tag{7}$$

where D is the domain of the tube cross section. The expression for the dimensionless integral-average heat flux takes the form

$$\frac{\tilde{q}_S R}{\lambda (T_w - T_0)} = -\text{Re}_* \sum_{k=1}^{2} a_k(Z,\ \text{Fo})\, \Phi_k(X,\ Y). \tag{8}$$

Equations (5) and (8) provide a means for analyzing the influence of the shape of the tube cross section on the heat-transfer process.

Figures 1a-c show the variation of the mass-average temperature of the fluid and the dimensionless integral-average heat flux at the tube wall for various values of the Prandtl and dynamic Reynolds numbers.

To test the validity of the solution obtained here we compare the results of calculations of the local Nusselt number Nu [1] for e = 0 (circular cross section) with the results of calculations of Nu according to the expression obtained in [1] for the steady-state problem in a circular tube. The comparison is made for Pr = 0.7 and Re = 10^4 and $5 \cdot 10^4$, where Re = $\bar{\omega} d/\nu$, $\bar{\omega}$ is the average velocity over the tube cross section, d is the tube diameter, $u_* = \bar{\omega}\sqrt{\xi/8}$ [1], and the frictional drag coefficient is calculated according to the formula of Filonenko [1]:

$$\xi = (1.82 \lg \text{Re} - 1.64)^{-2}.$$

It is seen in Fig. 2 that the discrepancy in the values of Nu is not greater than 3%, indicating that the second approximation already yields good agreement of the present results with the published data for the given special case of the problem.

LITERATURE CITED

1. B. S. Petukhov, L. G. Genin, and S. A. Kovalev, Heat Transfer in Nuclear Power Plants [in Russian], Atomizdat, Moscow (1974).
2. N. M. Galin, V. Yu. Dem'yanenko, V. M. Esin, and G. I. Maslenko, "Approximate expressions describing the distribution of the velocity profile and turbulent viscosity coefficient in incompressible flow for ducts of complex cross section," Tr. Mosk. Energet. Inst., No. 235 (1975).
3. P. V. Tsoi, Methods for the Solution of Special Heat- and Mass-Transfer Problems [in Russian], Énergiya, Moscow (1971).
4. S. G. Mikhlin, Variational Methods in Mathematical Physics [in Russian], Nauka, Moscow (1970).
5. R. Courant and D. Hilbert, Methods of Mathematical Physics, Vol. 2: Partial Differential Equations, Wiley, New York (1962).
6. A. A. Kochubei and A. A. Ryadno, "A method for the calculation of nonsteady convective heat transfer in noncircular ducts," in: Heat Physics and Heat Engineering [in Russian], No. 33 (1977).
7. N. M. Belyaev, A. A. Kochubei, and A. A. Ryadno, "Calculation of nonsteady heat transfer in a tube of elliptical cross section for laminar incompressible flow," Teplofiz. Vys. Temp., 15, No. 2 (1977).
8. H. Reichardt, Z. Angew. Math. Mech., 30, No. 7 (1951).